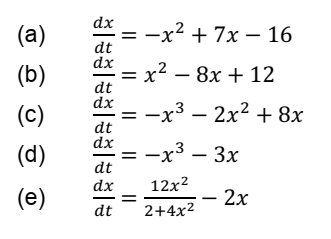
*Adaptive Filters 2019-20 – ASSIGNMENTS*

# Dynamic Systems

For each exercise, explain in detail how you get to your answer!

**DS1**. Study the listed differential equations by answering the following questions:

* Draw the phase portrait.
* How many equilibria are there and at which *x*?
* For each equilibrium find out whether is it stable or non-stable.
* Determine the final value of *x* when 𝑡→∞. (e.g., *x* converges to equilibrium, or *x* goes to infinity, etc.)
* List attractor/attractors and determine their basin/basins of attraction.



**DS2**. The following equation describes the production of a product, the concentration of which is denoted by x:



where *c* is a parameter representing a chemical used in the production. The initial state of the system is *x* = 0, and the value of *c* = 0. Then, at some point in time the value of *c* is slowly increased from *c* = 0 to *c* = *c*max and subsequently slowly decreased back to the value *c* = 0.

(a) Identify the state variable/variables of this system.

(b) Draw the graph of 𝑓(𝑥)=−𝑥(𝑥−0.3)(𝑥−1)+𝑐 for *c* = 0.

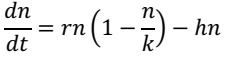
(c) Find the minimum value of 𝑓(𝑥)=−𝑥(𝑥−0.3)(𝑥−1)+𝑐 with *c* = 0.

(d) What will be the value of the concentration *x* of the product at the end of the described process for *c*max = 0.005?

(e) The same for *c*max = 0.02?

(f) Show (graphically) that there is a critical value of *c*max that separates the qualitatively different possible outcomes of this process. Find this critical value of *c*max. What is this type of critical value called?

**DS3**. Consider a model population with logistic growth which is subject to harvesting at a rate ℎ∙𝑛 which is proportional to the population size *n*:



Find the maximal yield.

**DS4**. From Chapter 9 (additional questions)

Sub section Ch4, Question 3

Sub section Ch 5, Question 1

# Preliminary to AF

**AF 1.** 10 minute presentation on

* 1. Gradient Descent
  2. Least Squares
  3. Wiener Filter

# Adaptive Filters

**AF2**. LMS on typical data. See lecture 5 slide 30

**AF3.** LMS and Normalised LMS

See Lecture 6, slide 15

* 1. Reproduce paragraph 6.7
  2. Chapter 6, problem 17

See Lecture 7, slide 13

* 1. Chapter 7, problem 10 (which continues from problem 6.17)

**AF4.** RLS Assignment

Problem 17 in Chapter 6, but now use RLS instead of LMS, as follows:

(a) is method independent, so could be answered.

(b) Explain why Problem 6.17.b is not applicable in case of RLS.

(c) Replace LMS by RLS.

(d) Replace LMS by RLS.

(e) Small step-size statistical theory does not apply to RLS, so instead of a theoretical comparison, show a comparison between the learning curves of LMS and RLS. Before plotting the curves, first predict which differences are likely to become visible, and explain your prediction. Then verify your prediction with the actual learning curves and write down your observations and conclusions.

**AF5.** Kalman Filter assignment

For this assignment, consider an old Volkswagen Beetle that is driving on a long straight road. Using a Kalman filter, we want to estimate the Beetle’s position on the road and the speed at which it is travelling. First based on speed measurements and then based on position measurements.

1. Assume that (in theory) the Beetle is travelling at a speed of 90 km/h.

Simulate velocity measurements by adding white noise (mean = 0 km/h, standard deviation = 20 km/h) to the theoretical speed and set the sample frequency to one sample per minute.

Design and build a Kalman filter (for example in Python or Octave/Matlab) to estimate the Beetle’s velocity and use the velocity estimates to derive its (one-dimensional) position on the road. (Assume at time 0 the Beetle is at km 0 along the road.) Present your results graphically and comment on them.

1. The Beetle’s owner recently bought a navigation system that uses GPS positioning. Assume again that the Beetle is travelling at a speed of 90 km/h and determine the theoretical positions of the car along the road at 1-minute intervals.

The road is passing through a forest, so the GPS measurements are rather noisy. Suppose that the GPS measurements give you the Beetle’s positions with additive white noise (mean = 0 m, standard deviation = 300 m) once per minute.

Design and build a Kalman filter to first estimate the Beetle’s position and then use the estimated positions to derive its velocity. (Again assume at time 0 the car is at km 0 along the road.)

Present your results graphically and comment on them.

Compare the new results with the results from (1) and comment on the differences (if any).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Assessment Aspect | Weight | Subgrade | Location of Proof | Insufficient (3) | Pass (5.5) | Good (8) | Excellent (10) |
| Model and simulate (sensor) systems using tools such as Modelica and pySimulator. | 0.25 |  | Assignments 3,4,5…. |  |  |  |  |
| Describe systems in the time-domain and/or frequency domain using stationary discrete-time stochastical processes and models, such as autoregressive and moving-average models. | 0.25 |  | NA |  |  |  |  |
| Describe systems using linear and nonlinear dynamical system models by applying techniques such as state space, phase portrait, linearization and bifurcations. | 0.25 |  | Assignment 1 | Has failed to answer most of the questions correctly. | Has approached and answered most of the questions posed correctly but additional evaluation would have been expected. | Has approached and answered all posed questions correctly and thoroughly. | Has included analysis of the answers, for example if they meet expectations, or what extra information the results give. |
| Clean data using advanced filter techniques, such as (normalized) Least-Mean-Square adaptive filters, Recursive Least Squares Adaptive Filters and Kalman filtering. | 0.25 |  | Assignments 3, 4, 5…. |  |  |  |  |
| Analyse sensor systems using model-based reasoning |  |  | NA |  |  |  |  |
| Represent data in a meaningful way. |  |  | NA |  |  |  |  |